July 20–21, 2026 • Koç University, İstanbul

7th Turkish Robotics Conference

Poster ID: 116

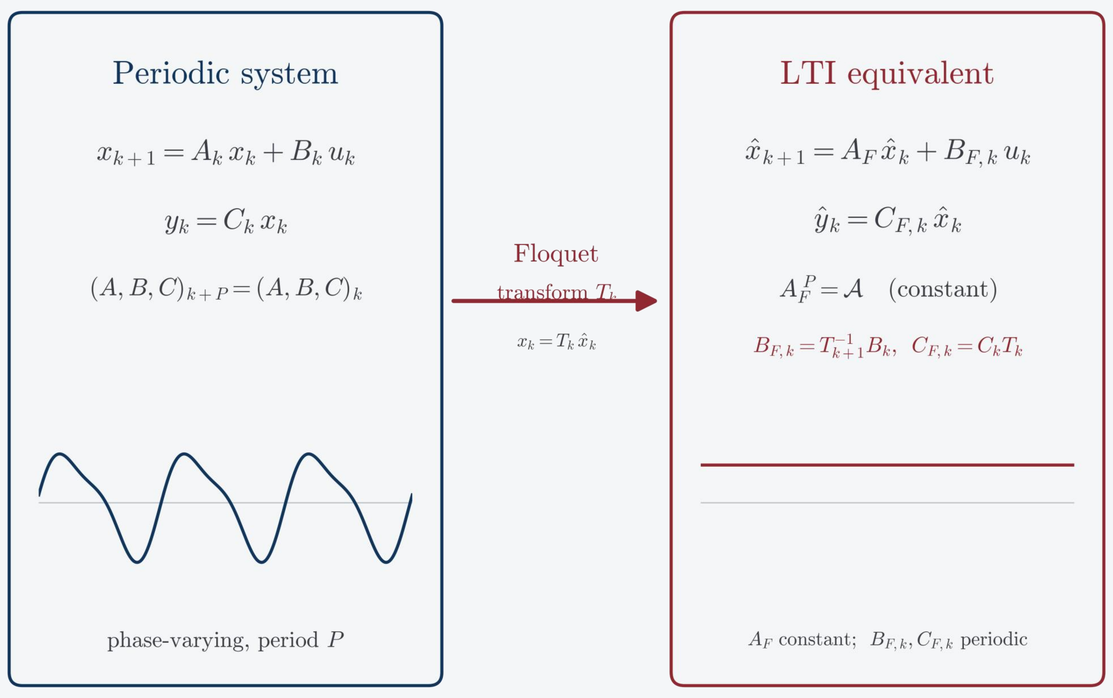
A Unified Framework for Identifying Floquet-Equivalent Models of Linear Discrete-Time Periodic Systems

Onurcan Yılmaz[1], İsmail Uyanık[1] • [1]Department of Electrical–Electronics Engineering, Hacettepe University, Ankara, Türkiye

Presenter: Onurcan Yılmaz • oyilmaz@ee.hacettepe.edu.tr

1 Introduction & Problem

- Linear time-periodic (LTP) systems arise naturally in robotics and biomechanics, wherever the governing dynamics repeat with the motion cycle.
- Floquet theory is the canonical tool for analyzing them: every LTP system admits a similarity transform into a linear time-invariant (LTI) equivalent.
- The LTI equivalent is governed by a constant state-transition matrix A_F, the principal P-th root of the monodromy matrix.
- This lets the full machinery of LTI analysis like stability or control design to be applied directly to periodic systems.
- Recovering it from data faces two numerical obstacles:
  - identifying the phase-dependent state-transition matrices $\{A_k, B_k\}$
  - reliably computing the monodromy's matrix root when the estimate is noisy or non-normal



Where LTP Dynamics Arise

- Legged locomotion and gait synchronization in humans and animals
- Rotating machinery, wind-turbine aerodynamics, and helicopter rotor blade-passing effects
- Power-converter switching cycles and periodically modulated communication channels

2 Method

2.1 Periodicity-Regularized Identification

- COSMIC [1] treats identification as a per-step local regression $x_{i+1} \approx A(i)x_i + B(i)u_i$, stacking $\{A(i), B(i)\}$ into a parameter block G_i at every time step i.
- Its baseline cost J_{COS} combines a data-fit term with a step-to-step smoothness penalty between consecutive parameter blocks, since phase-dependent dynamics change gradually.
- Our contribution: a periodicity regularizer J_{per} ties together every parameter block that shares the same phase k across all N cycles — directly enforcing $A_{k+P}=A_k$ on the estimator itself.
- The combined cost $J_{total}=J_{COS}+J_{per}$ stays quadratic, so the original COSMIC solver applies unchanged — just with a denser, phase-coupled normal-equation structure.
- Solving still leaves per-step estimates $\{A(i), B(i)\}$ that vary slightly across cycles due to noise; a final cycle-averaging step collapses the N repetitions of each phase into the periodic pair used for the Floquet analysis.

Local regression at step i: $x_{i+1} \approx A(i)\,x_i + B(i)\,u_i, \quad G_i \leftarrow \{A(i),\, B(i)\}$

Baseline (COSMIC)

$$J_{\mathrm{COS}} = \sum_{i=1}^{T-1} \|A(i)x_i + B(i)u_i - x_{i+1}\|_2^2 + \lambda \sum_{i=2}^{T-1} \|G_i - G_{i-1}\|_F^2 \quad \text{(step-to-step smoothness)}$$

Periodicity regularizer - our contribution

$$J_{\mathrm{per}} = \lambda_p \sum_{k=1}^{P} \sum_{i=1}^{N-1} \|G_{k+iP} - G_{k+(i-1)P}\|_F^2$$

ties parameter blocks that share phase k across all N cycles

$$J_{\mathrm{total}} = J_{\mathrm{COS}} + J_{\mathrm{per}}$$

remains quadratic, sparse block-banded normal equations

2.2 Robust Floquet Reduction

- Prior approaches [2,3] extract the principal P-th root from the eigenstructure of the monodromy's Schur factor, unreliable for near-defective matrices.
- We replace this with the Schur–Padé algorithm [4,5], approximating the root via a rational Padé approximant on the scaled triangular factor.
- This sidesteps eigendecomposition entirely.
- The result: substantially better stability on the noisy, near-defective monodromies that data-driven identification produces.

Input-output trajectory
$\{u_k,\, x_k\}_{k=0}^{T-1}$

Periodicity-Regularized COSMIC
$J_{\mathrm{total}} = J_{\mathrm{COS}} + J_{\mathrm{per}}$

Cycle-Averaging
$\hat{A}_k, \hat{B}_k = \frac{1}{N} \sum_{i=0}^{N-1} A_{k+iP},\, B_{k+iP}$

Monodromy estimate
$\hat{\mathcal{A}} = \hat{A}_{P-1} \cdots \hat{A}_0$

Schur-Pade root
$A_F = Q\,T^{1/P}\,Q^*$

Floquet-equivalent model
$\{A_F,\, T_k,\, B_{F,k},\, C_{F,k}\}$

Simulation Setup

- Trajectories of length $T = 100P + 1$ samples (10 periods per trial), broadband i.i.d. Gaussian input
- Process noise with covariance $\Sigma_w = 0.1 \cdot I_n$ added to the state
- 1,000 independent random LDTP systems per (n, P) configuration

3 Key Findings

- Validated via a Monte Carlo study over 50,000 randomly generated stable LDTP systems (order n = 2–6, period P = 2–100).
- The periodicity-regularized identifier reaches a 100% success rate across every tested configuration.
- NRMSE stays below 0.09 across all configurations, identification is robust and largely insensitive to system order.
- The Schur–Padé reduction yields highly accurate Floquet-equivalent trajectories.
- Prediction error follows a U-shaped curve with a minimum near $P \approx$ 27–42 (Fig. 1), confirming the pipeline is reliable across a broad operating range.
- At very long periods the Floquet success rate itself declines (10–40% by P = 100 for higher-order systems) due to the growth of the Schur–Padé residual, not the periodic identification, is the limiting factor.

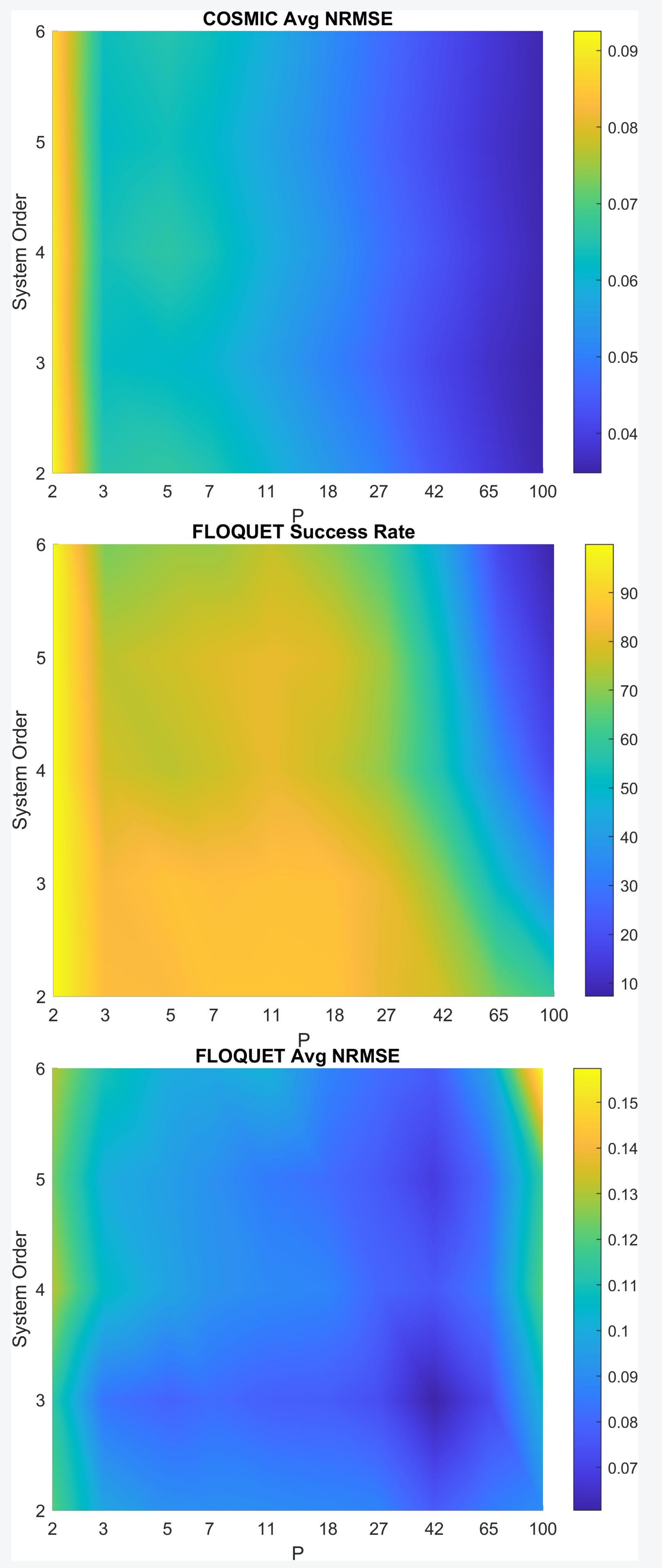


Fig. 1. Monte Carlo validation (50,000 trials): (a) average prediction NRMSE of the COSMIC estimation model; (b) Floquet reduction success rate over system order n and period P; (c) average prediction NRMSE of the Floquet-equivalent model, with a U-shaped minimum near P ≈ 27–42.

References

[1] Carvalho et al., arXiv:2112.04355 (2021).
[2] Van Dooren & Sreedhar, Linear Algebra Appl. 212 (1994).
[3] Sreedhar & Van Dooren, Proc. CISS (1993).
[4] Higham & Lin, SIAM J. Matrix Anal. Appl. 32 (2011).
[5] Higham & Lin, SIAM J. Matrix Anal. Appl. 34 (2013).

This research was supported by TÜBİTAK under Grant No. 124E232 awarded to Dr. İsmail Uyanık and 2211A Scholarship awarded to Onurcan YILMAZ.

HU NeuRo Lab

HACETTEPE ÜNİVERSİTESİ